

11. (a) (i) Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$. (8)

that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$.

(ii) The following table gives the variations of a periodic current over a period.

| | | | | | | | |
|------------|------|-------|-------|-------|--------|--------|------|
| t secs : | 0 | $T/6$ | $T/3$ | $T/2$ | $2T/3$ | $5T/6$ | T |
| A amps : | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

By harmonic analysis, show that there is a direct current part of 0.75 amps in the variable current. Also obtain the amplitude of the first harmonic. (8)

Or

(b) (i) Find the half range sine series for $f(x) = \sin ax$ in $(0, l)$. (8)

(ii) Find the complex form of the Fourier series of e^{-ax} , $-l < x < l$. Deduce that when α is constant other than an integer

$$\cos \alpha x = \sin \alpha l \sum_{n=-\infty}^{\infty} \frac{\alpha l}{\alpha^2 l^2 - n^2 \pi^2} (-1)^n e^{in\pi/l}. \quad (8)$$

12. (a) (i) Show that the Fourier transform of $f(x) = \begin{cases} a - |x|, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ is

$$\sqrt{\frac{2}{\pi}} \left(\frac{1 - \cos as}{s^2} \right). \text{ Hence deduce that } \int_0^{\infty} \left(\frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}. \quad (8)$$

(ii) Solve for $f(x)$, the integral equation

$$\int_0^{\infty} f(x) \sin sx \, dx = \begin{cases} 1, & 0 \leq s < 1 \\ 2, & 1 \leq s < 2 \\ 0, & s \geq 2 \end{cases} \quad (8)$$

Or

(b) (i) Find the Fourier transform of $e^{-|x|}$ and hence deduce that

$$\int_0^{\infty} \frac{\cos xt}{1+t^2} dt = \frac{\pi}{2} e^{-|x|}. \quad (8)$$

(ii) Prove that $F_C[x f(x)] = \frac{d}{ds} [F_S\{f(x)\}]$ and $F_S[x f(x)] = -\frac{d}{ds} [F_C\{f(x)\}]$ (8)

13. (a) (i) Form the PDE by eliminating the arbitrary functions f_1, f_2 from relation $z = x f_1(x+t) + f_2(x+t)$. (8)

(ii) Solve $\left(\frac{p}{2} + x\right)^2 + \left(\frac{q}{2} + y\right)^2 = 1$. (8)

Or

(b) (i) Solve $x^2 p + y^2 q = z(x+y)$. (8)

(ii) Solve $(r+s-6t) = y \cos x$. (8)

14. (a) An uniform elastic string of length 60 cms is subjected to a constant tension of 2 Kg. If the ends fixed and the initial displacement $y(x,0) = 60x - x^2$, $0 < x < 60$, while the initial velocity is zero, find the displacement function $y(x,t)$. (16)

Or

(b) Solve the problem of heat conduction in a rod given that the temperature function $u(x,t)$ is subject to the condition, $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $0 \leq x \leq l$, $t > 0$

(i) u is finite as $t \rightarrow \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for $x=0$ and $x=l$, $t > 0$

(iii) $u = lx - x^2$ for $t=0$, $0 \leq x \leq l$. (16)

15. (a) (i) Find $Z(r^n \sin n\theta)$, $Z^{-1}\left[\frac{z}{z^2 + 4z + 3}\right]$. (4+4)

(ii) Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem. (8)

Or

(b) (i) Using complex residue theorem evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$. (8)

(ii) Solve using Z-transforms technique the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$. (8)