Reg. No. : [| | | | | | | | | | | |]

Question Paper Code : 91576

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find a particular integral of the differential equation $(D^2+6D+5)y = e^{-5x}$.

2. Transform the differential equation $x^2y'' - xy' + 2y = 0$ with constant coefficients.

3. Find $\nabla(\nabla.((x^2-yz)\bar{i}+(y^2-xz)\bar{j}+(z^2-xy)\bar{k}))$ at the point $(1, -1, 2)$.

4. State Green's theorem in the plane.

5. Give an example of a complex-valued function which is differentiable at a point but not analytic at that point.

6. If $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$, verify whether u is harmonic.

7. State Cauchy's integral formula.

8. Find the residue of $\left\{\frac{\sin 3z}{z^6}\right\}$ at $z = 0$.

9. State sufficient conditions for the existence of Laplace transform.

10. State final value theorem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the differential equation $y'' + a^2 y = \tan ax$ by variation of parameters method. (8)

(ii) Solve the following simultaneous differential equations.

$$\frac{dx}{dt} + 2x - 3y = t \text{ and } \frac{dy}{dt} - 3x + 2y = e^{2t}.$$ (8)

Or

(b) (i) Solve $((x+1)^2 D^2 + (x+1) D + 1) y = 4 \cos \log(x+1)$. (8)

(ii) Solve $(D^3 - 7D - 6) y = (1+x) e^{2x}$. (8)

12. (a) (i) Prove that $div(\phi \bar{F}) = \phi \, div \, \bar{F} + \nabla\phi \cdot \bar{F}$. Also, determine the value of n for which $r^n \bar{R}$ is solenoidal, where $\bar{R} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\bar{R}|$. (8)

(ii) Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ over the volume of the cuboid formed by the planes $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$. (8)

Or

(b) (i) Prove that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2 z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. (8)

(ii) Verify Stokes' theorem for $\vec{F} = (x^2 + y^2)\vec{i} + 2xy\vec{j}$ where S is the rectangle in the xy-plane formed by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$. (8)

13. (a) (i) If $f = u + iv$ is an analytic function, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$ (8)

(ii) Find the bilinear transformation which maps the points $\infty, 2, -1$ to $1, \infty$ and 0 respectively. (8)

Or

$u(x,y) = e^{2x}(x \sin 2y + y \cos 2y)$. (8)

(ii) If $f = u + iv$ is analytic on a domain D and $|f|$ is a constant on D, prove that f must be a constant on D. (8)

14. (a) (i) If $F(a) = \oint_C \frac{3z^2 + 7z + 1}{z - a} dz$ where $C : |z| = 2$ and $|a| \neq 2$, find $F(3)$ and $F''(1-i)$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$ by the method of contour integration, if a and b are positive. (8)

Or

(b) (i) Find the Laurent's series of $f(z) = \frac{3z - 2}{z(z^2 - 4)}$ valid in the region $2 < |z + 2| < 4$. (8)

(ii) Using contour integration method show that $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ if $a > b > 0$. (8)

15. (a) (i) (1) Find the Laplace transform of $f(t) = \frac{\sin^2 t}{t}$. (4)

(2) Find the value of $\int_0^{\infty} te^{-3t} \cos 2t \, dt$. (4)

(ii) Solve $y'' + 9y = \cos 2t$ given that $y(0) = 1$ and $y(\pi/2) = -1$, by the method of Laplace transform. (8)

Or

(b) (i) (1) Find $L^{-1}\left(\log \frac{s^2 + 1}{s(s+1)}\right)$. (4)

(2) Using convolution theorem, find y if $L(y) = \frac{s}{(s^2 + a^2)^2}$. (4)

(ii) Find $L(f(t))$ if $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & if \ a \le t \le 2a \end{cases}$ and $f(t + 2a) = f(t)$. (8)
